Analytic thermoelectric couple modeling: variable material properties and transient operation

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Thermoelectricity

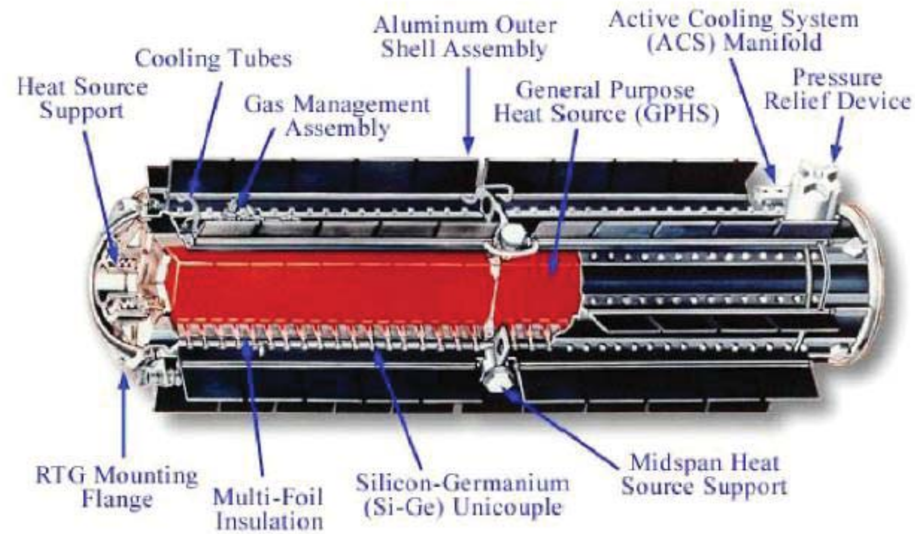
- Study of the coupled transport of electrical and thermal energy.
- Solid-state phenomenon requires no moving parts or working fluids, and generates no noise, torque, or vibrations.
  - As a result thermoelectric devices are extremely reliable.
- Power Generation
  - Spacecraft, automotive, aerospace, gas pipelines, well sites, and offshore platforms.
- Refrigeration
  - On chip cooling, electronics, and automotive.
- **High reliability, low conversion efficiency.**

Spacecraft Power

- Radioisotope thermoelectric generators (RTG) have powered 45 spacecraft.
  - Voyager (1977), Ulysses (1990), Cassini (1997), New Horizons (2006), and Curiosity (2011).

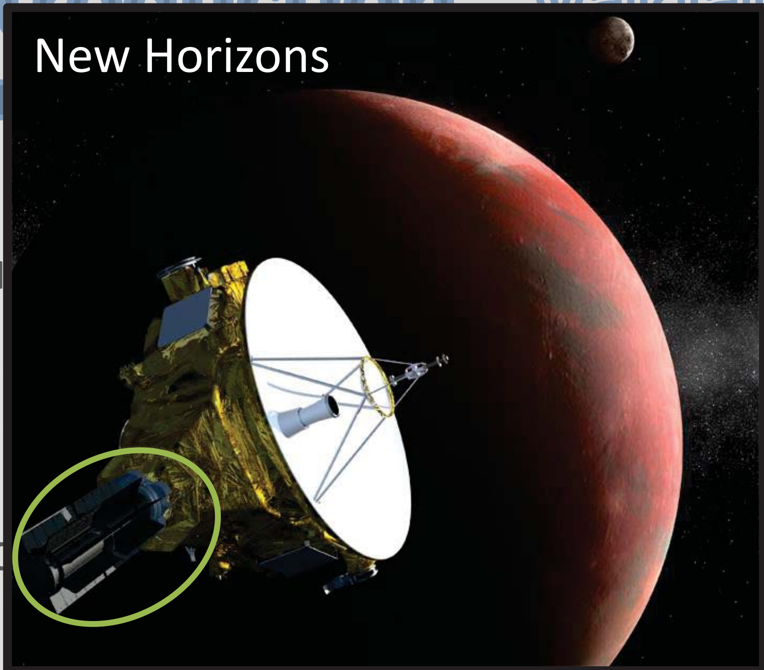
Lange et al. Energy Conversion and Management **49** (2008) 391-401.

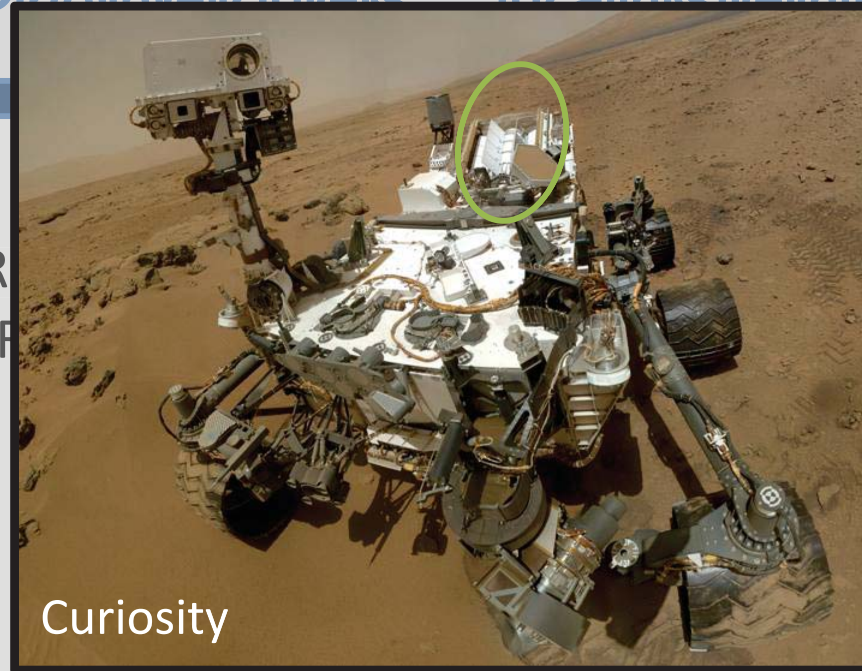
GPHS-RTG (Galileo/Ulysses)



Bennett et al. AIP Proceedings **969** (2008) 663-671.

Introduction – Variable Properties – Transient





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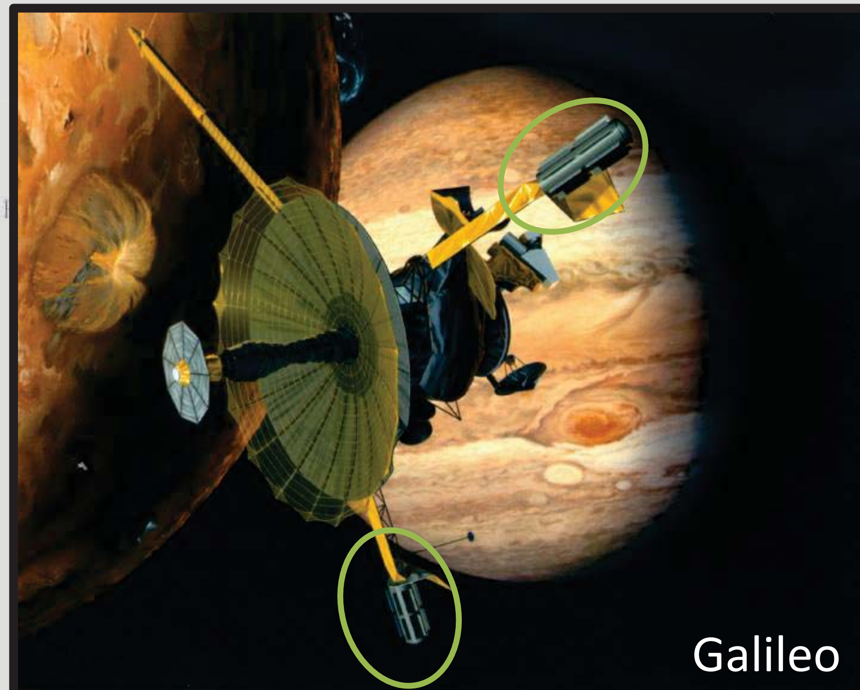

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Photos nasa.gov

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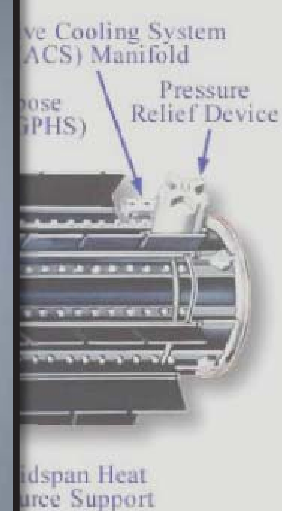

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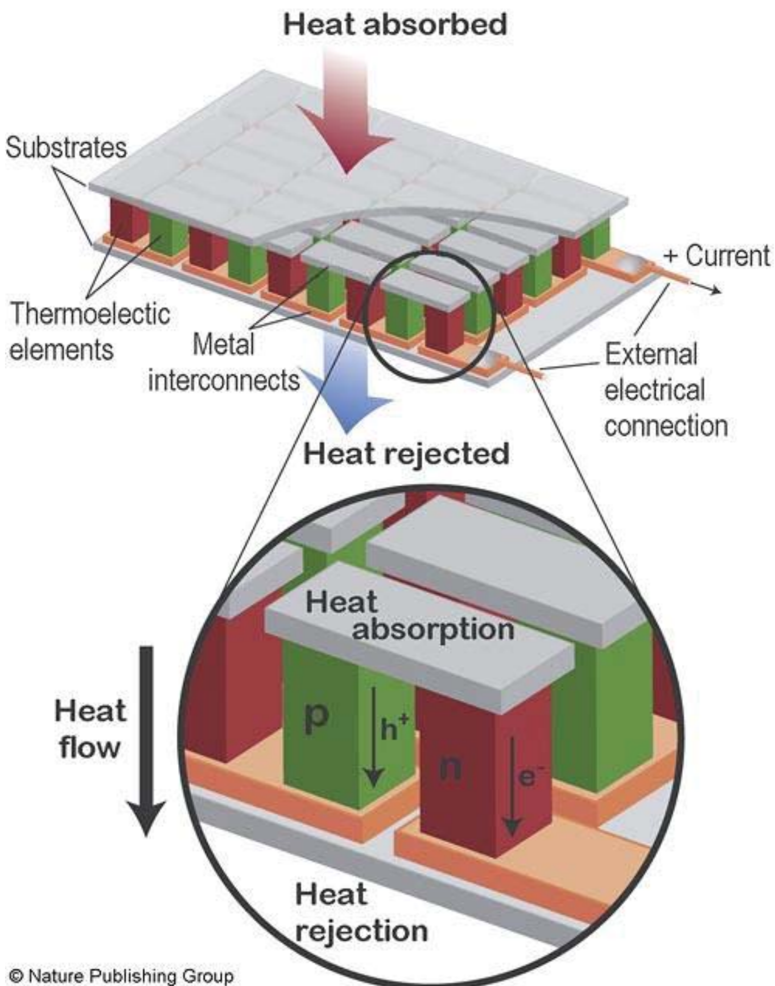
Thermocouple



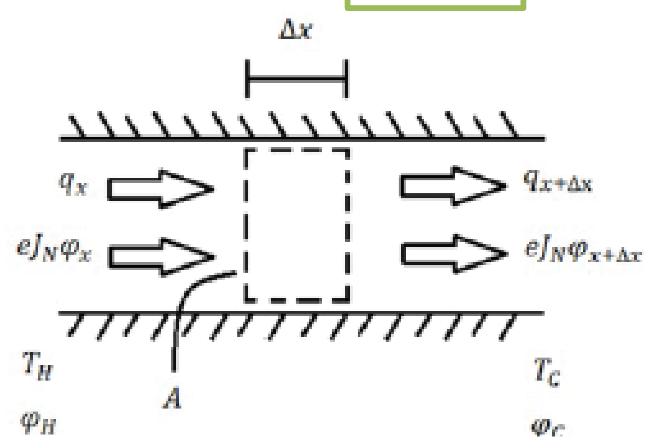
$$\eta = \frac{P_{out}}{Q_{in}}$$

Irreversible Thermodynamics

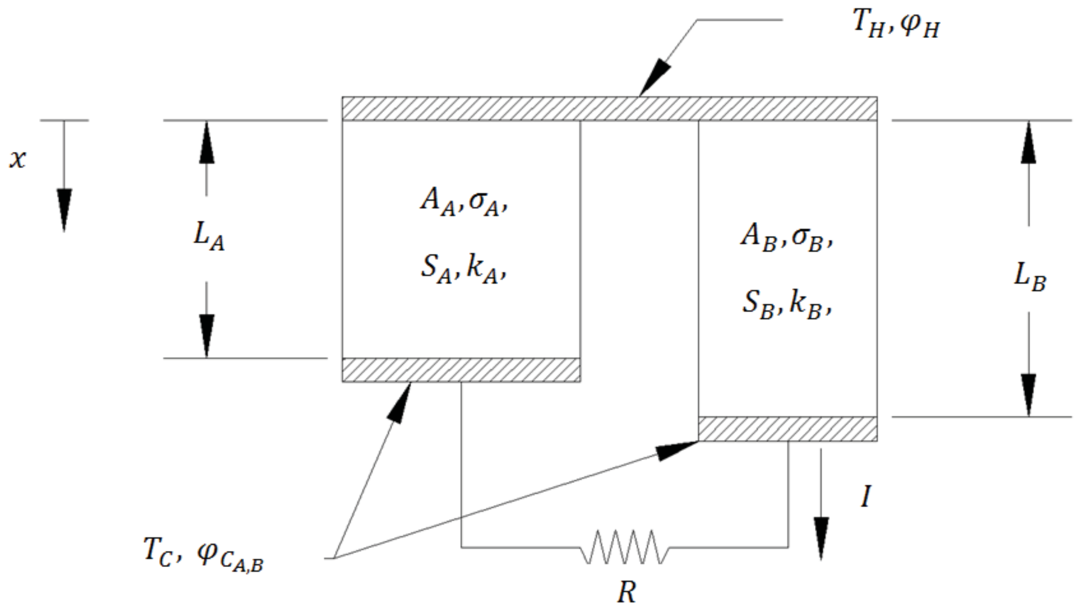
- 1931 Lars Onsager discussed coupled irreversible processes to unify thermoelectric phenomena into a single study.
- Study results in two transport laws for a thermoelectric conductor.

Ohm's Law- $$eJ_N = -\sigma\frac{d\phi}{dx}\boxed{-S\sigma\frac{dT}{dx},}$$

Fourier's Law- $$q'' = \boxed{STeJ_N} - k\frac{dT}{dx}.$$



Classic Model



Thermal- $$\frac{d}{dx}\left[-k_{A,B}\frac{dT_{A,B}}{dx}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{dT_{A,B}}{dx} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = 0$$

Electrical- $$\frac{d\varphi_{A,B}}{dx} = -S_{A,B}\frac{dT_{A,B}}{dx} - \frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$$

System- $$\varphi_B(L_B) - \varphi_A(L_A) = IR$$

Classic Parameters

Geometric- $$X = \frac{A_B L_A}{A_A L_B}$$

Load- $$Y = \frac{R}{\frac{L_B}{\sigma_B A_B} + \frac{L_A}{\sigma_A A_A}}$$

Materials- $$Z(X) = \frac{(S_B - S_A)^2}{\left(\frac{1}{\sigma_A} + \frac{1}{\sigma_B X}\right)(k_A + k_B X)}$$

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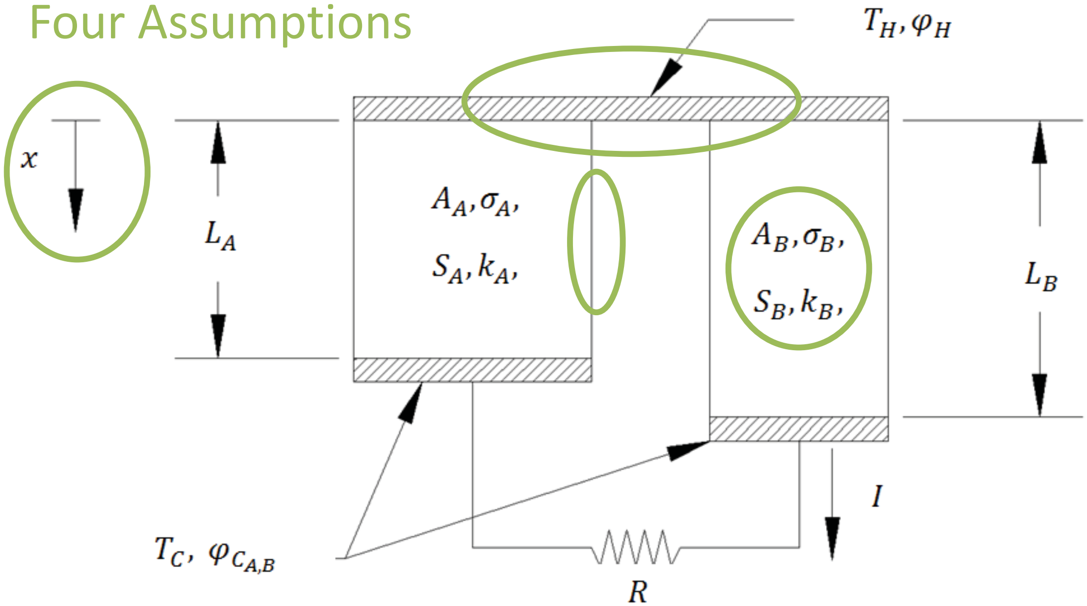
Classic Model

Four Assumptions



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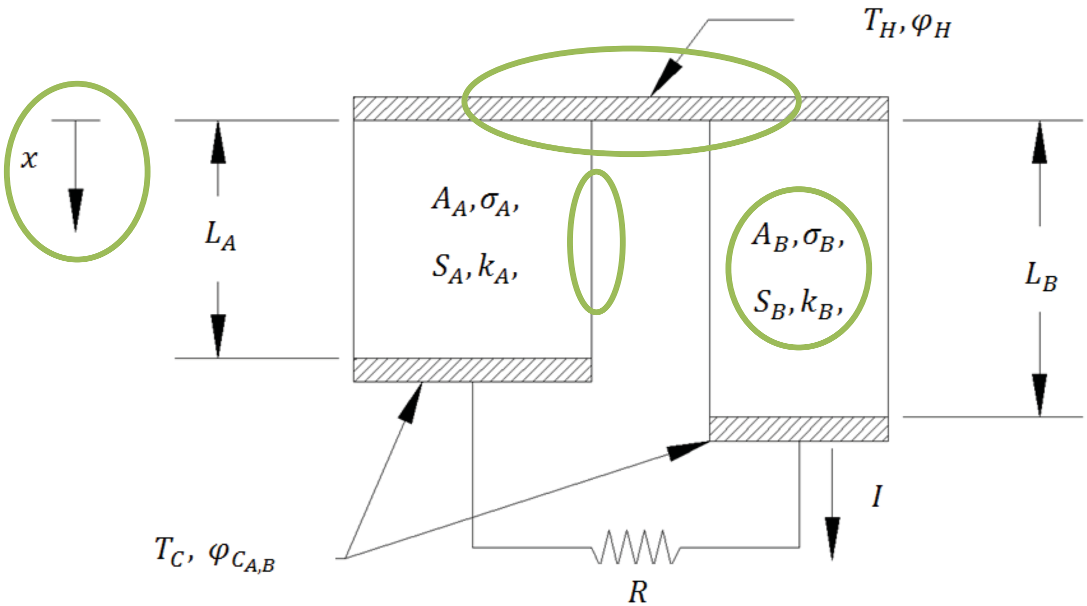

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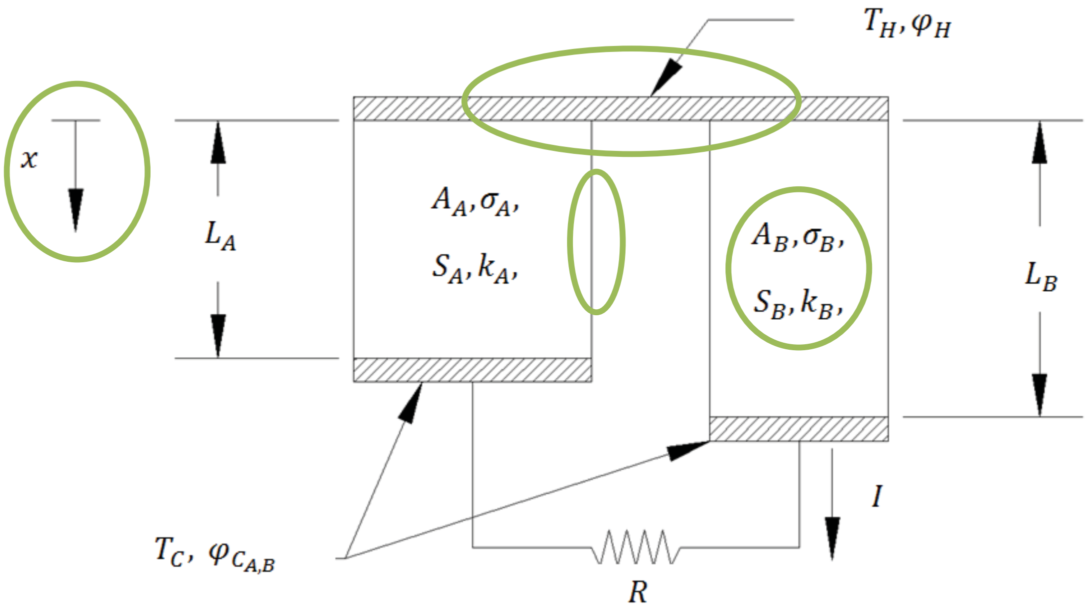

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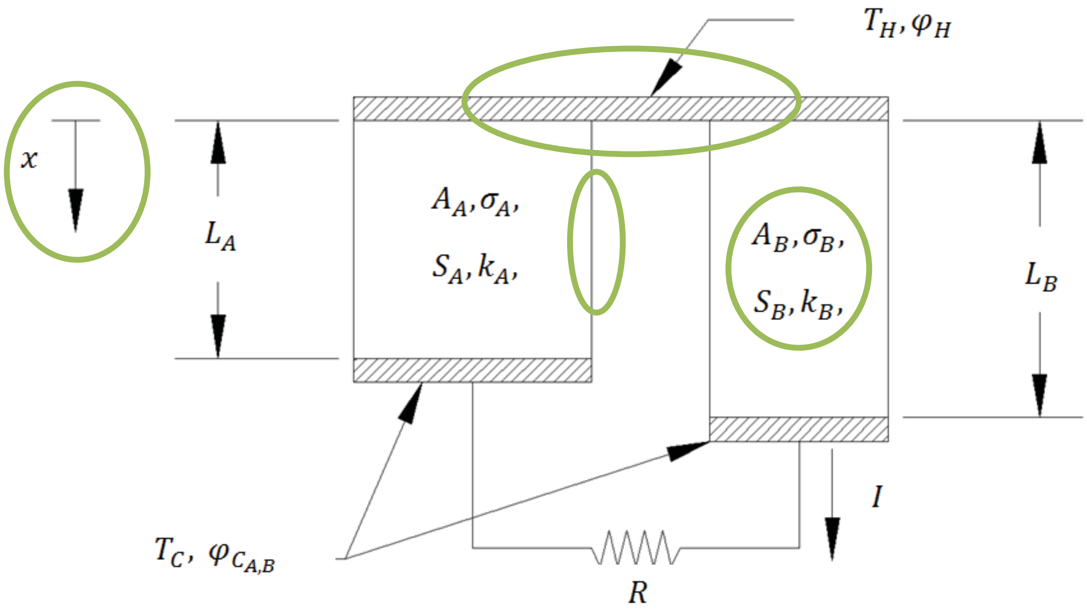

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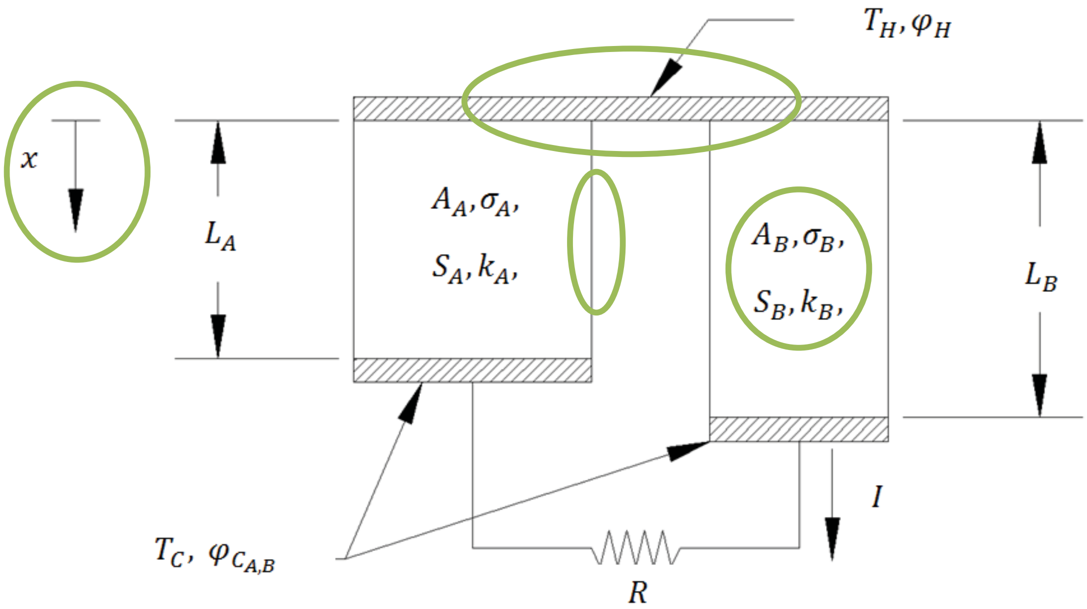

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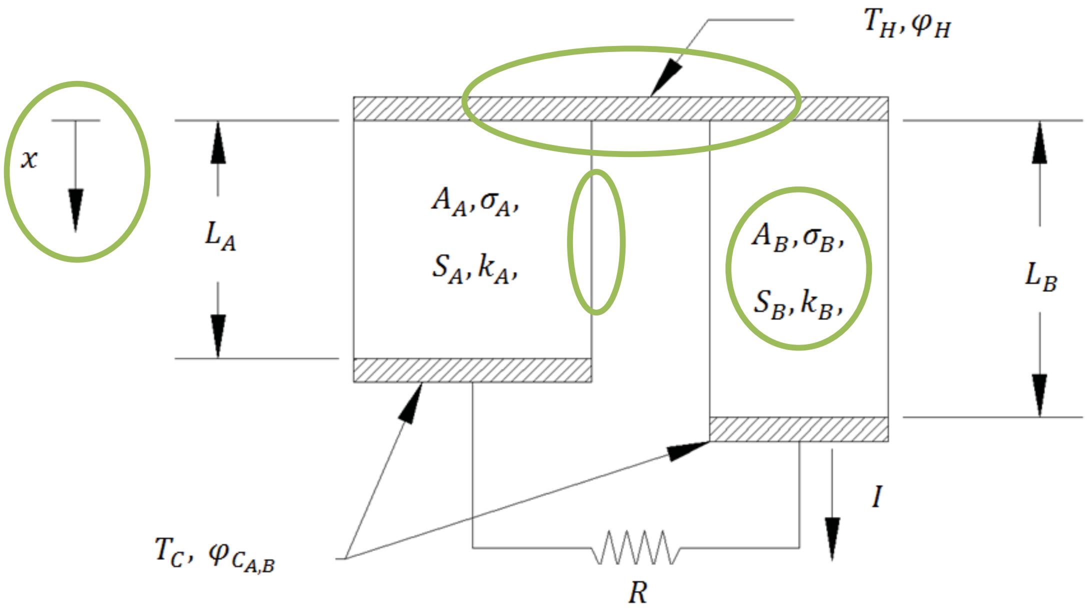

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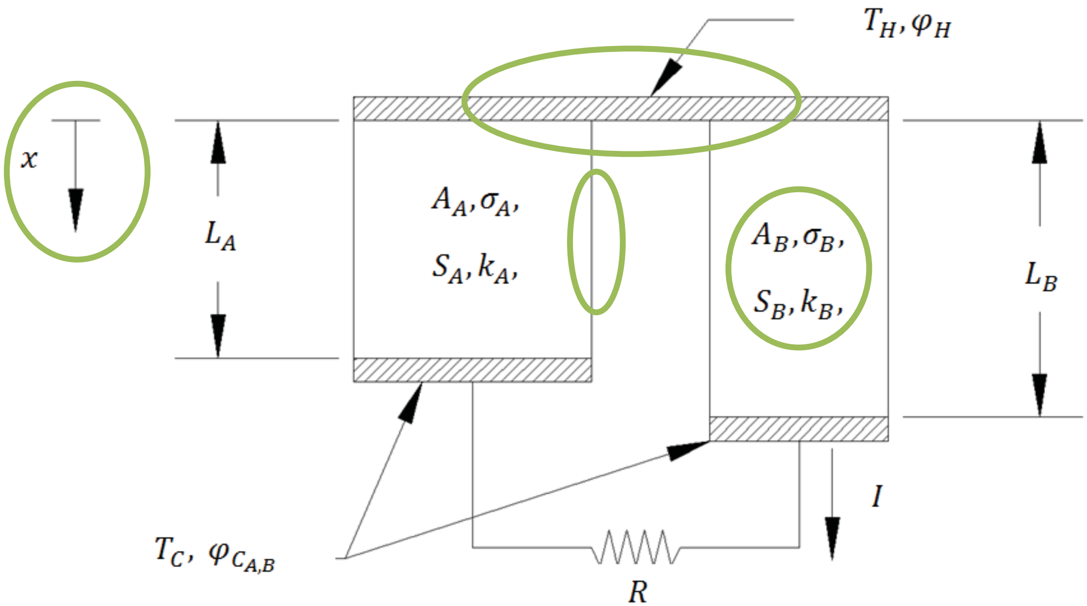

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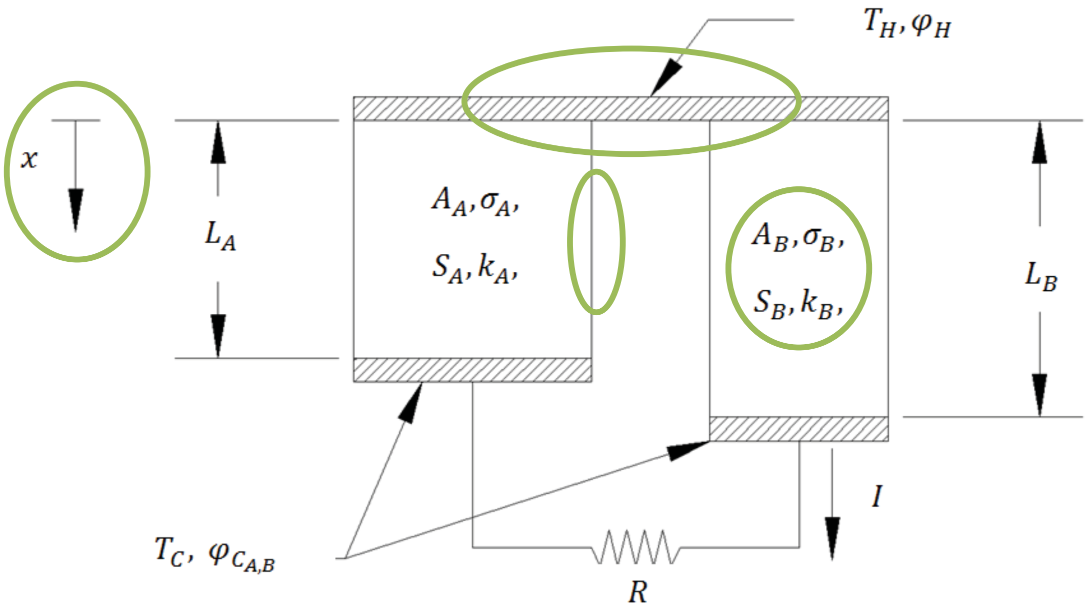

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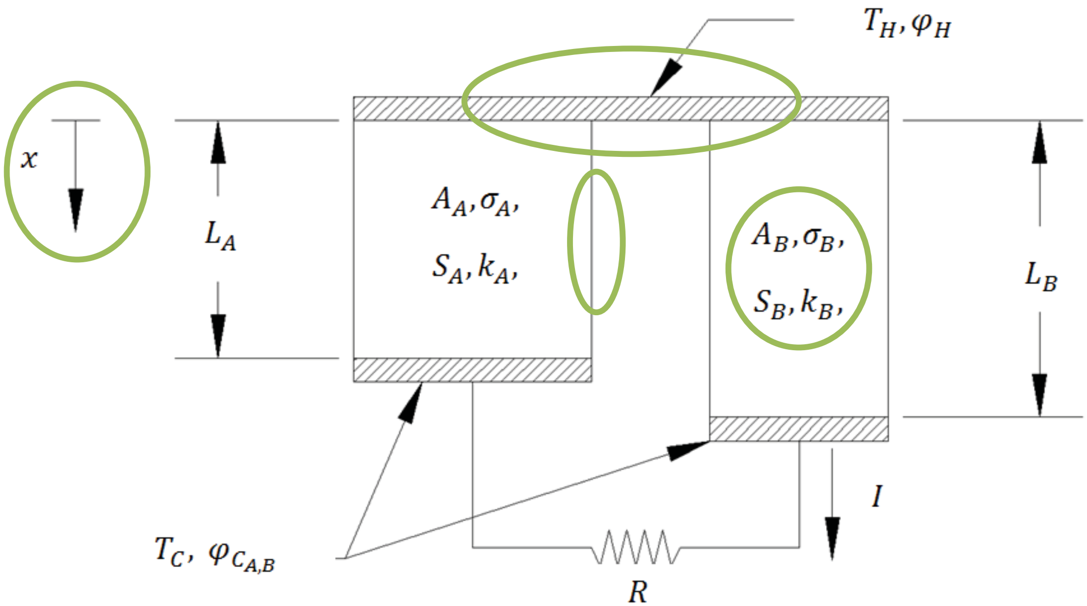

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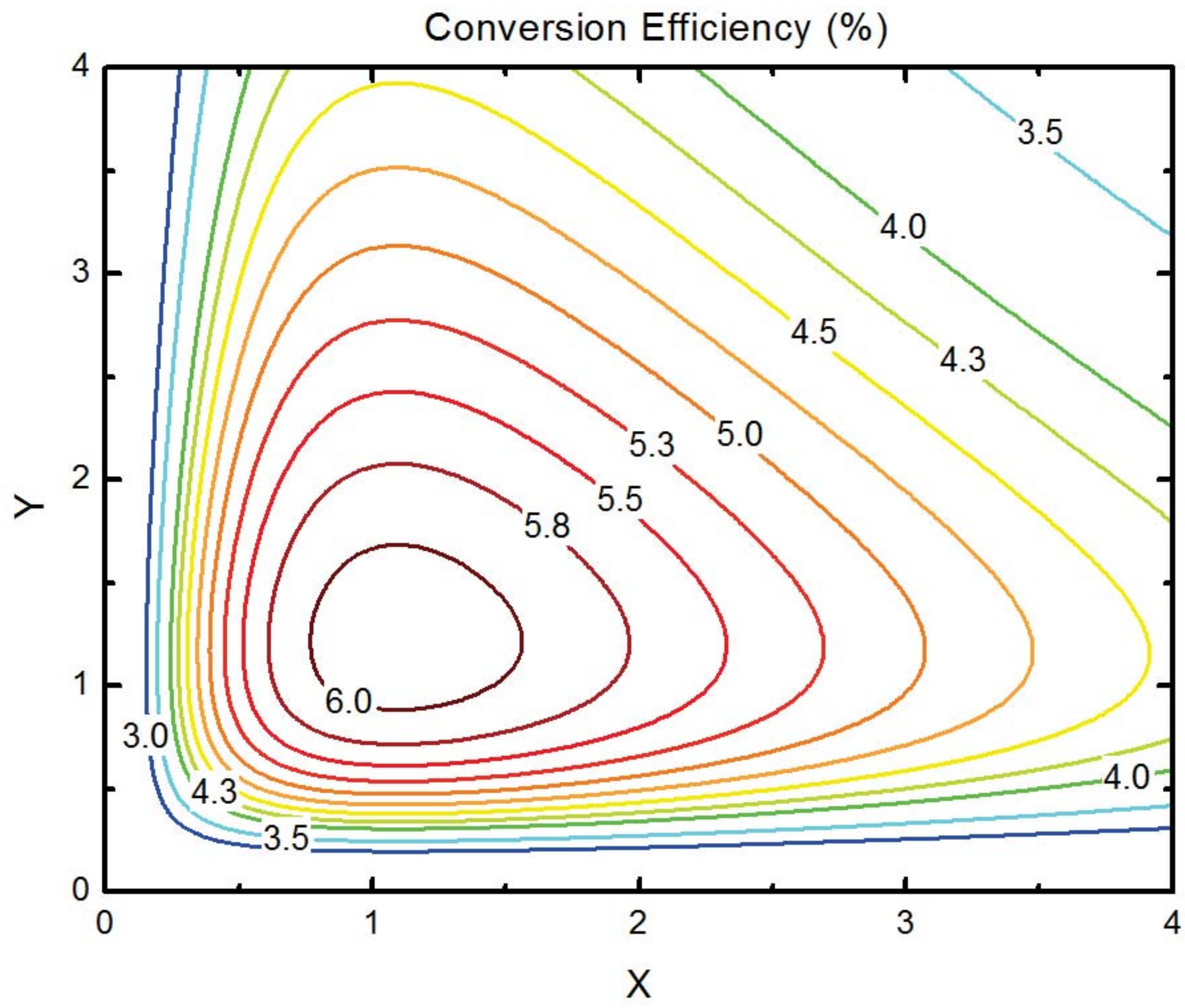

Mackey et al. Applied Energy **134** (2014) 374-381.

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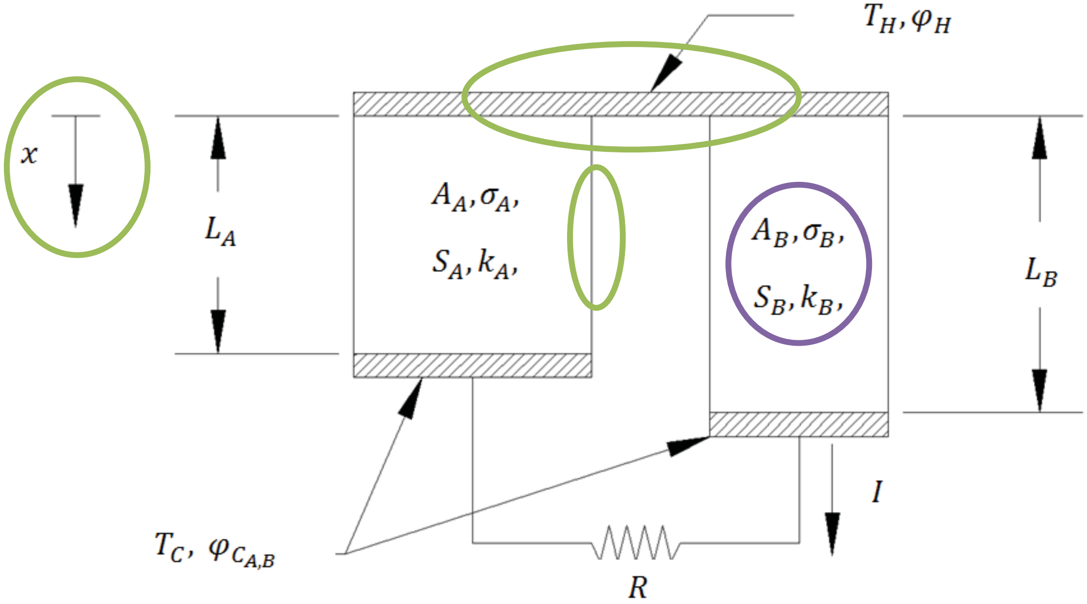
Variable Properties Model



Material Properties by Asymptotic Expansion-

$$\frac{1}{\sigma(T)} = \rho(T) = \tilde{\rho}\frac{\rho(T)}{\tilde{\rho}} = \tilde{\rho}\left(\rho_0 + \epsilon\rho_1\Delta T\hat{T}\right)$$

$$S(T) = \tilde{S}\frac{S(T)}{\tilde{S}} = \tilde{S}\left(S_0 + \epsilon S_1\Delta T\hat{T}\right)$$

$$k(T) = \tilde{k}\frac{k(T)}{\tilde{k}} = \tilde{k}\left(k_0 + \epsilon k_1\Delta T\hat{T}\right)$$

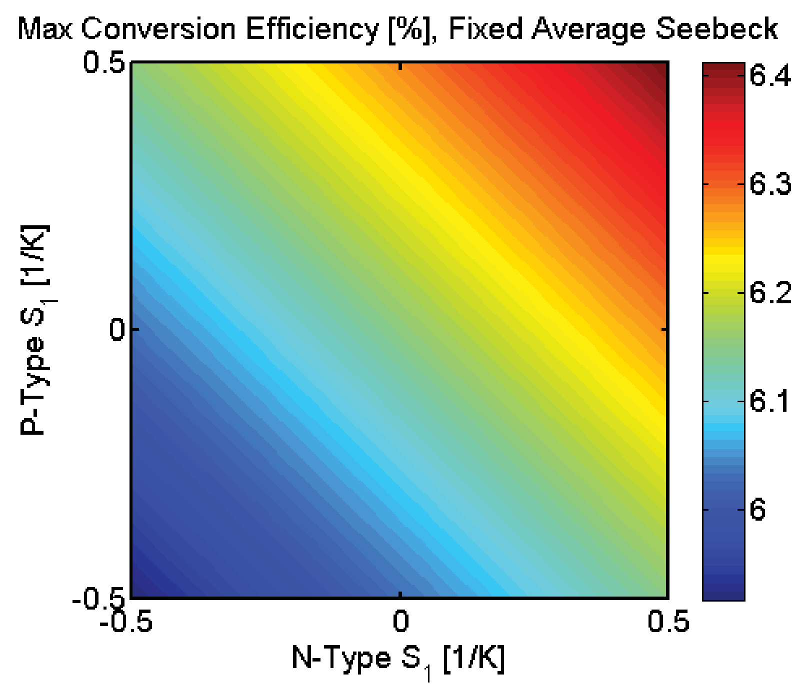
Asymptotic Expansion

$$\hat{T} = \frac{T}{\Delta T} \qquad \hat{\varphi} = \frac{\varphi}{\Delta S\Delta T} \qquad \hat{I} = \frac{IR}{\Delta S\Delta T}$$

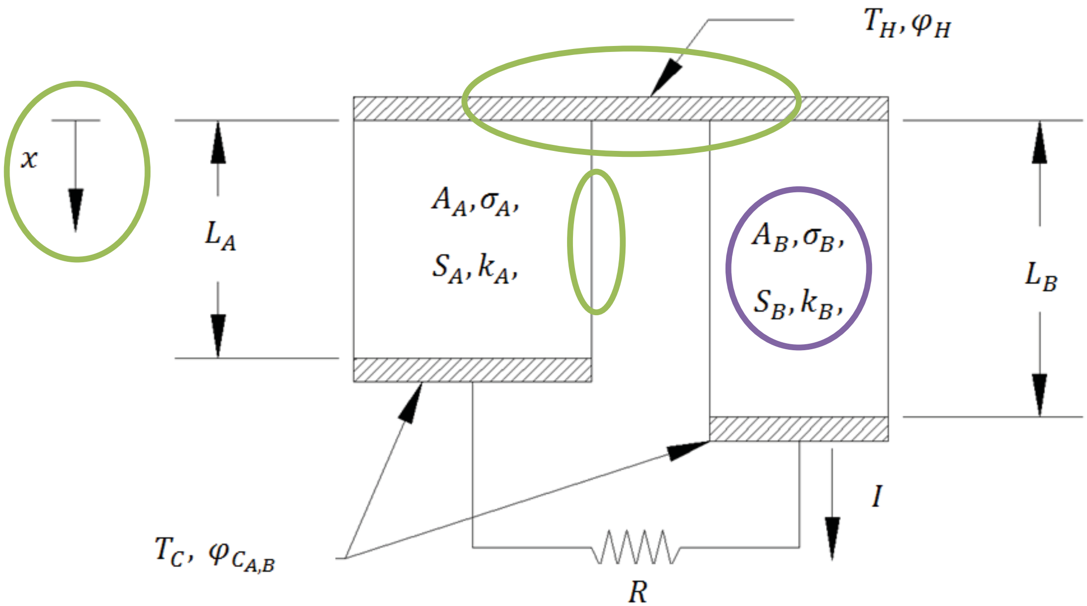
$$\hat{T} = T_0 + \epsilon T_1$$

$$\hat{\varphi} = \varphi_0 + \epsilon\varphi_1$$

Variable Seebeck



Variable Properties Model



Material Properties by Asymptotic Expansion-

$$\frac{1}{\sigma(T)} = \rho(T) = \tilde{\rho}\frac{\rho(T)}{\tilde{\rho}} = \tilde{\rho}\left(\rho_0 + \epsilon\rho_1\Delta T\hat{T}\right)$$

$$S(T) = \tilde{S}\frac{S(T)}{\tilde{S}} = \tilde{S}\left(S_0 + \epsilon S_1\Delta T\hat{T}\right)$$

$$k(T) = \tilde{k}\frac{k(T)}{\tilde{k}} = \tilde{k}\left(k_0 + \epsilon k_1\Delta T\hat{T}\right)$$

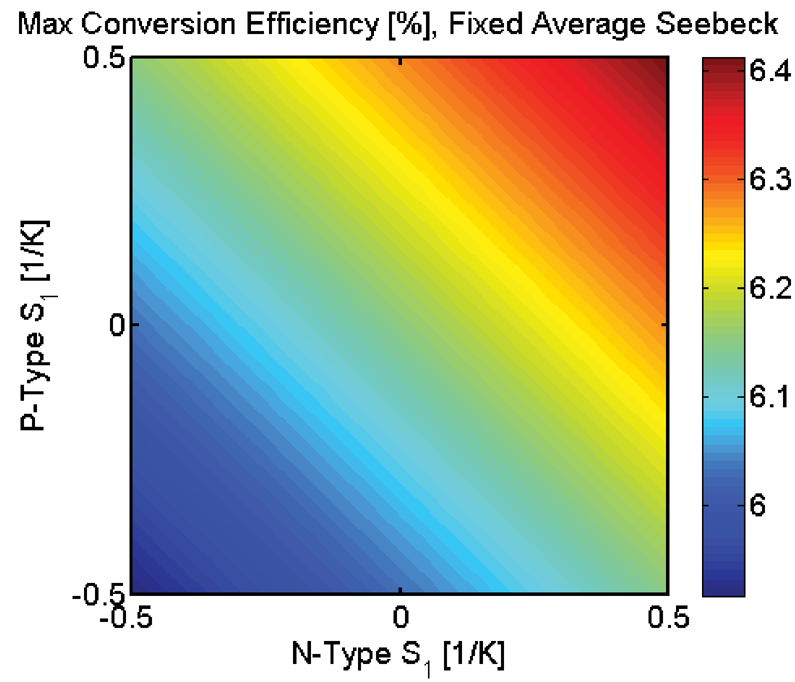
Asymptotic Expansion

$$\hat{T} = \frac{T}{\Delta T} \qquad \hat{\varphi} = \frac{\varphi}{\Delta S\Delta T} \qquad \hat{I} = \frac{IR}{\Delta S\Delta T}$$

$$\hat{T} = T_0 + \epsilon T_1$$

$$\hat{\varphi} = \varphi_0 + \epsilon\varphi_1$$

Variable Seebeck



Material Properties

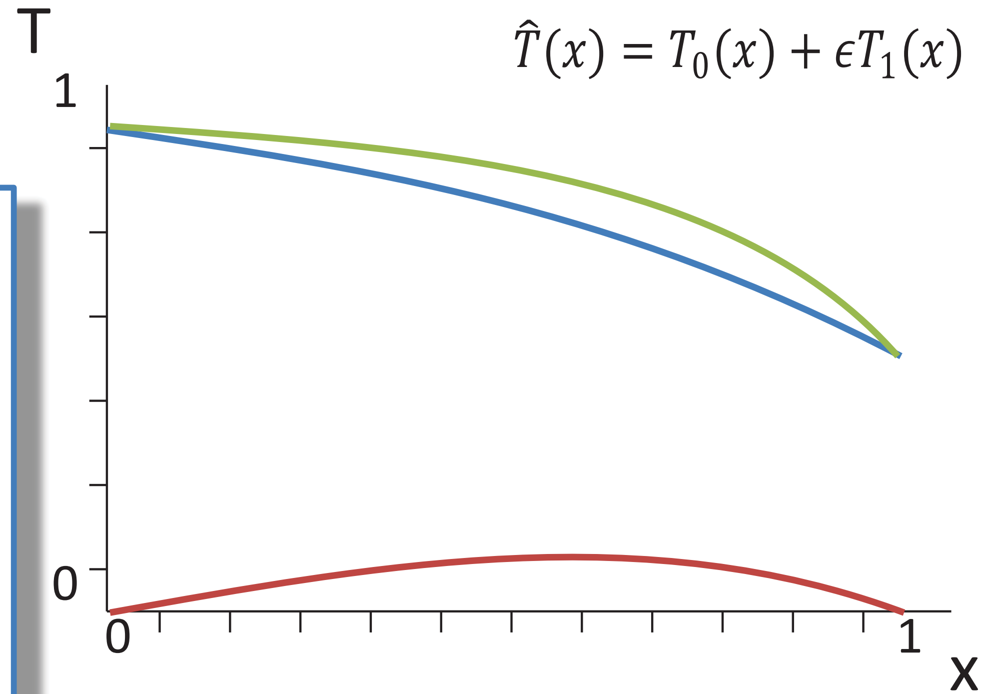
$$\rho(T) = \tilde{\rho}\frac{\rho(T)}{\tilde{\rho}} = \tilde{\rho}\left(\rho_0 + \epsilon\rho_1\Delta T\hat{T}\right)$$

$$S(T) = \tilde{S}\frac{S(T)}{\tilde{S}} = \tilde{S}\left(S_0 + \epsilon S_1\Delta T\hat{T}\right)$$

$$k(T) = \tilde{k}\frac{k(T)}{\tilde{k}} = \tilde{k}\left(k_0 + \epsilon k_1\Delta T\hat{T}\right)$$

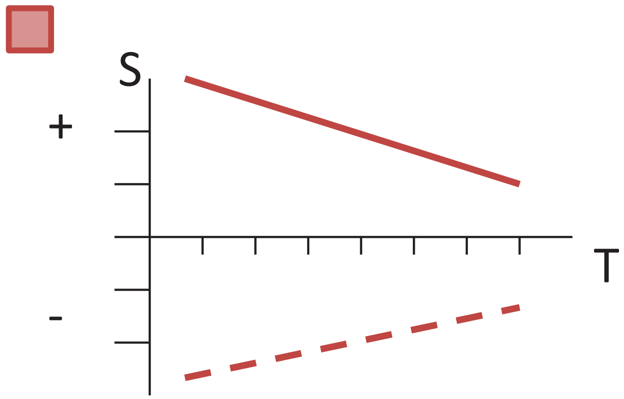
Asymptotic Expansion Method

$$\hat{T}(x) = T_0(x) + \epsilon T_1(x)$$

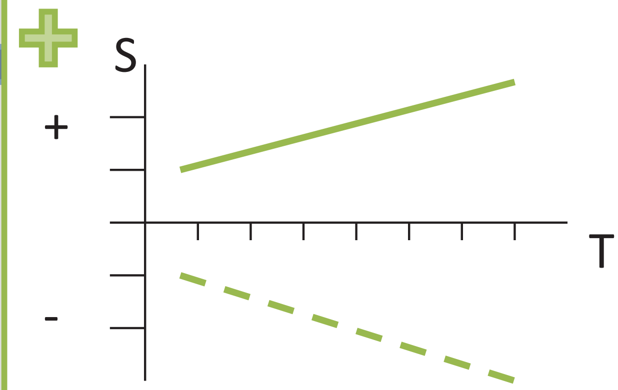


$$\frac{d}{dx}\left[-k_{A,B}\frac{dT_{A,B}}{dx}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{dT_{A,B}}{dx} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = 0$$

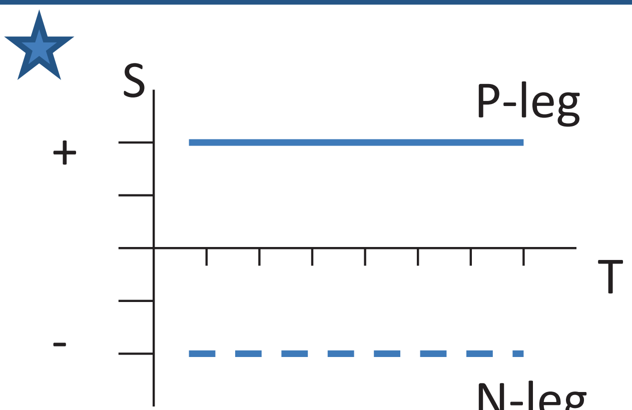
- Leading order temperature solution
- First order temperature correction
- Combined temperature solution



Conversion Efficiency 6.00%



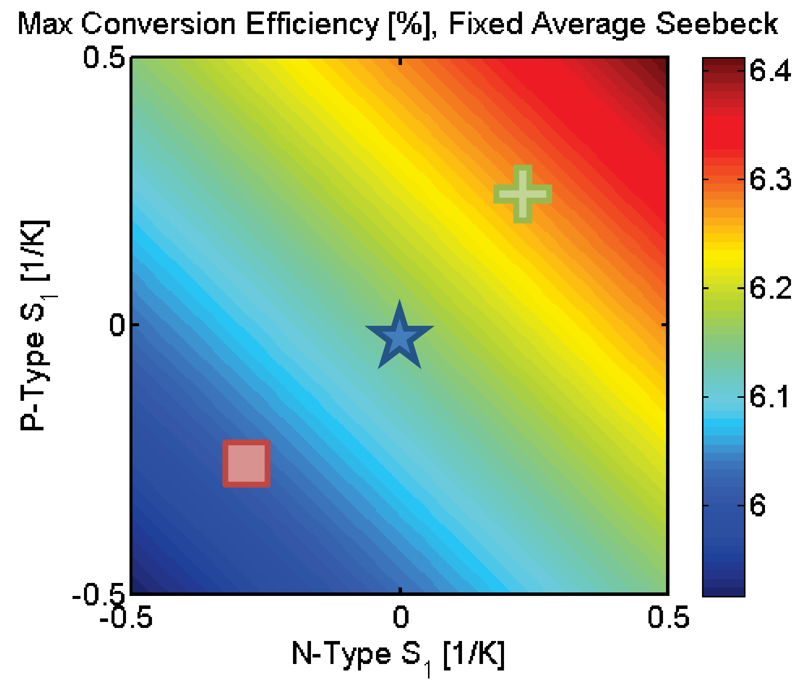
Conversion Efficiency 6.30%

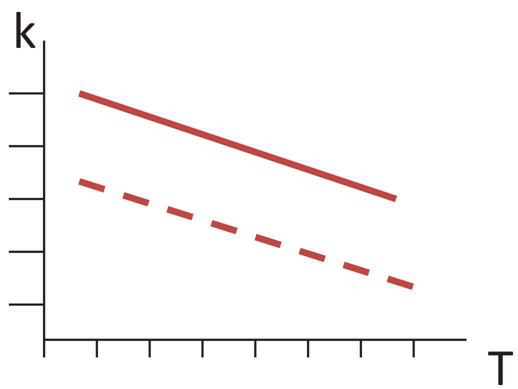


Conversion Efficiency 6.15%

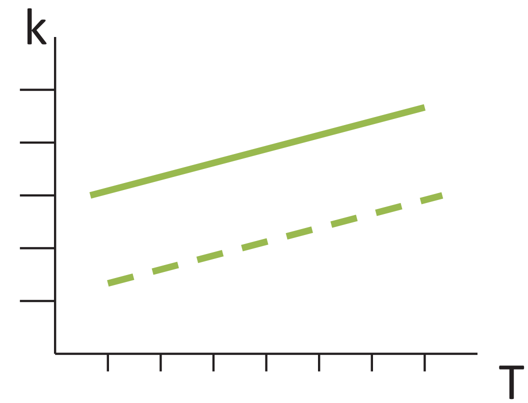
$$S(T) = \tilde{S}\frac{S(T)}{\tilde{S}} = \tilde{S}\left(S_0 + \epsilon S_1 \Delta T \hat{T}\right)$$

Variable Seebeck

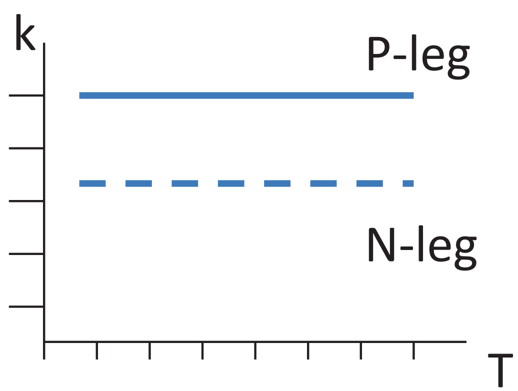




Conversion Efficiency 5.84%



Conversion Efficiency 6.69%

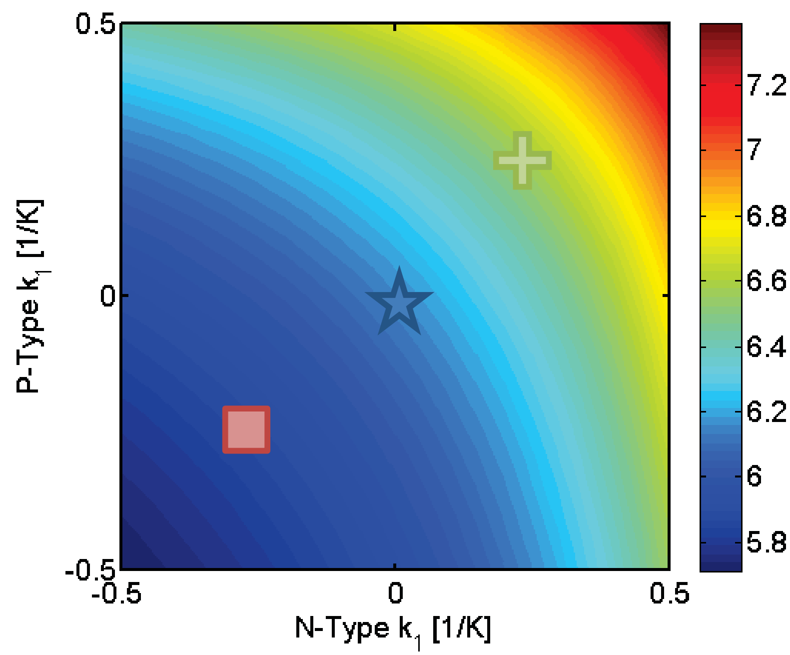


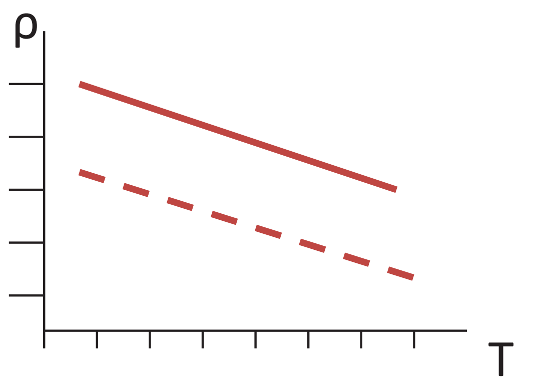
Conversion Efficiency 6.15%

$$k(T) = \tilde{k}\frac{k(T)}{\tilde{k}} = \tilde{k}\left(k_0 + \epsilon k_1 \Delta T \hat{T}\right)$$

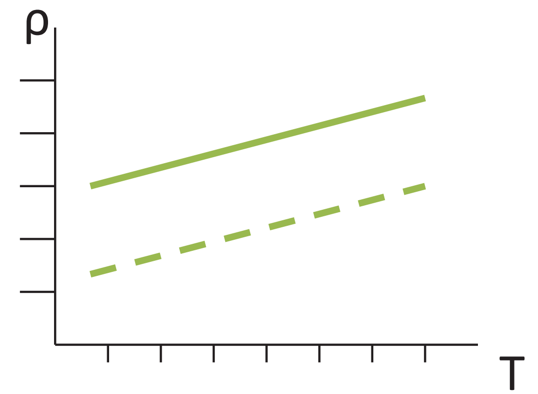
Variable Thermal

Max Conversion Efficiency [%], Fixed Average Thermal Cond.

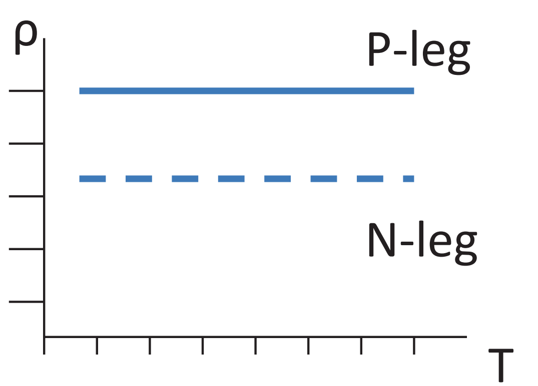




Conversion Efficiency 6.20%



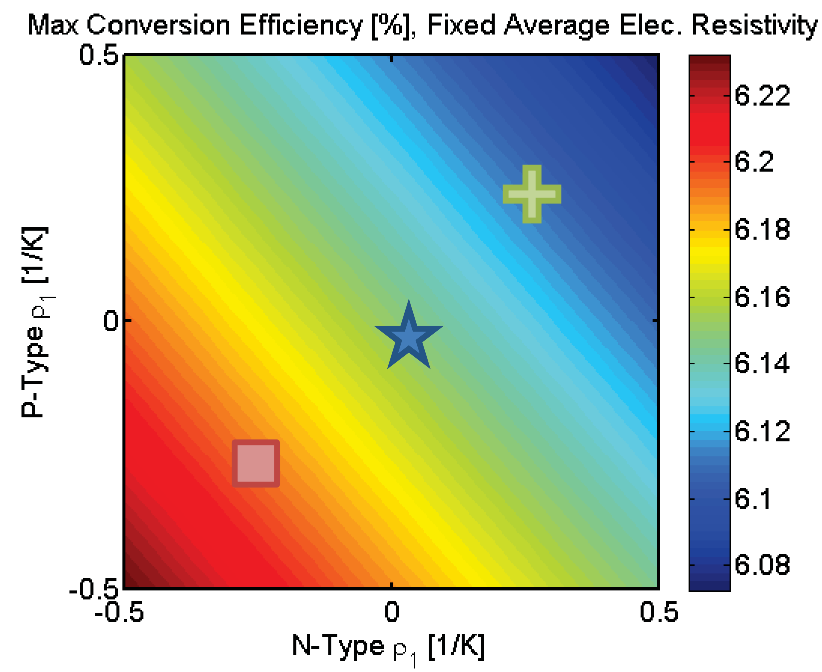
Conversion Efficiency 6.10%



Conversion Efficiency 6.15%

$$\rho(T) = \tilde{\rho}\frac{\rho(T)}{\tilde{\rho}} = \tilde{\rho}(\rho_0 + \epsilon\rho_1 \Delta T \hat{T})$$

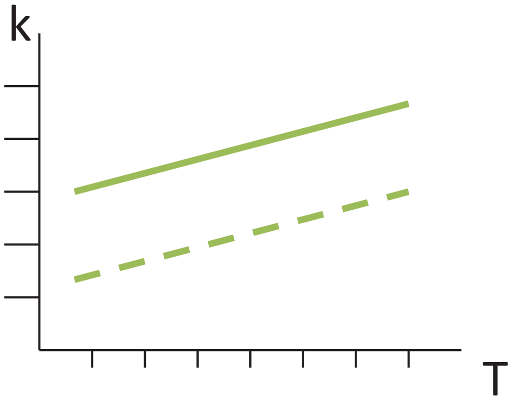
Variable Resistivity



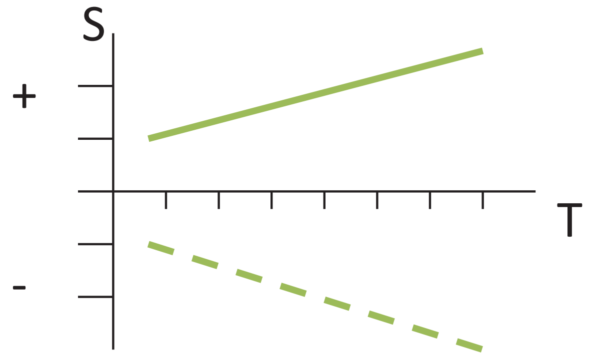
Variable Property Model Summary

Material Property	Temperature Dependence	Conversion Efficiency	Sensitivity (K)
Thermal Conductivity	↑	↑	0.60
Absolute Seebeck Coefficient	↑	↑	0.25
Electrical Resistivity	↑	↓	0.08

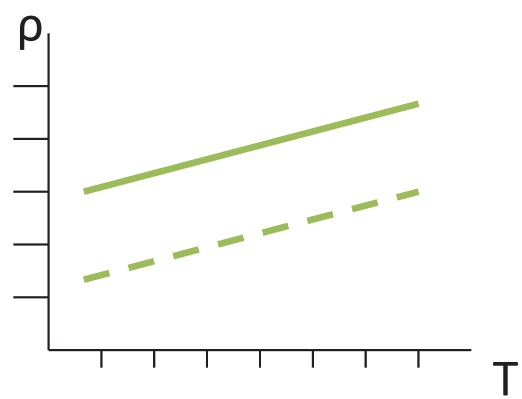
$$Sensitivity = \frac{\Delta\eta}{\Delta(S_1)}$$



Conversion Efficiency 6.69%

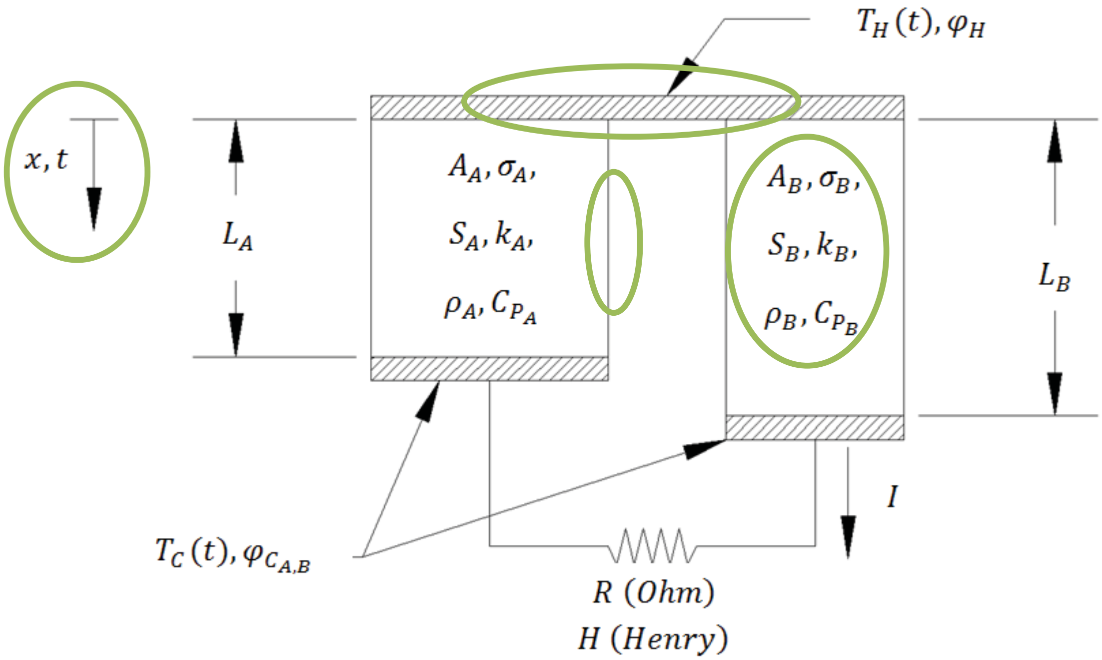


Conversion Efficiency 6.30%



Conversion Efficiency 6.10%

Transient Model

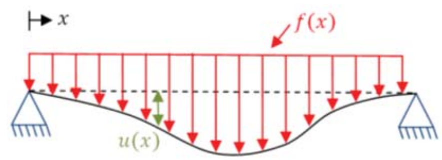


Thermal- $$\frac{\partial}{\partial x}\left[-k_{A,B}\frac{\partial T_{A,B}}{\partial x}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{\partial T_{A,B}}{\partial x} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = \rho_{A,B}c_{p_{A,B}}\frac{\partial T_{A,B}}{\partial t}$$

Electrical- $$\frac{\partial \varphi_{A,B}}{\partial x} = -S_{A,B}\frac{\partial T_{A,B}}{\partial x} - \frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$$

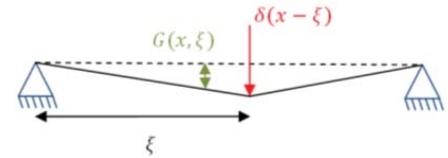
System- $$\varphi_B(L_B) - \varphi_A(L_A) = IR + H\frac{dI}{dt}$$

Green's Function Solution



$$\mathcal{L}u(x) = f(x)$$

$$\mathcal{L}^*G(x,\xi) = \delta(x-\xi)$$



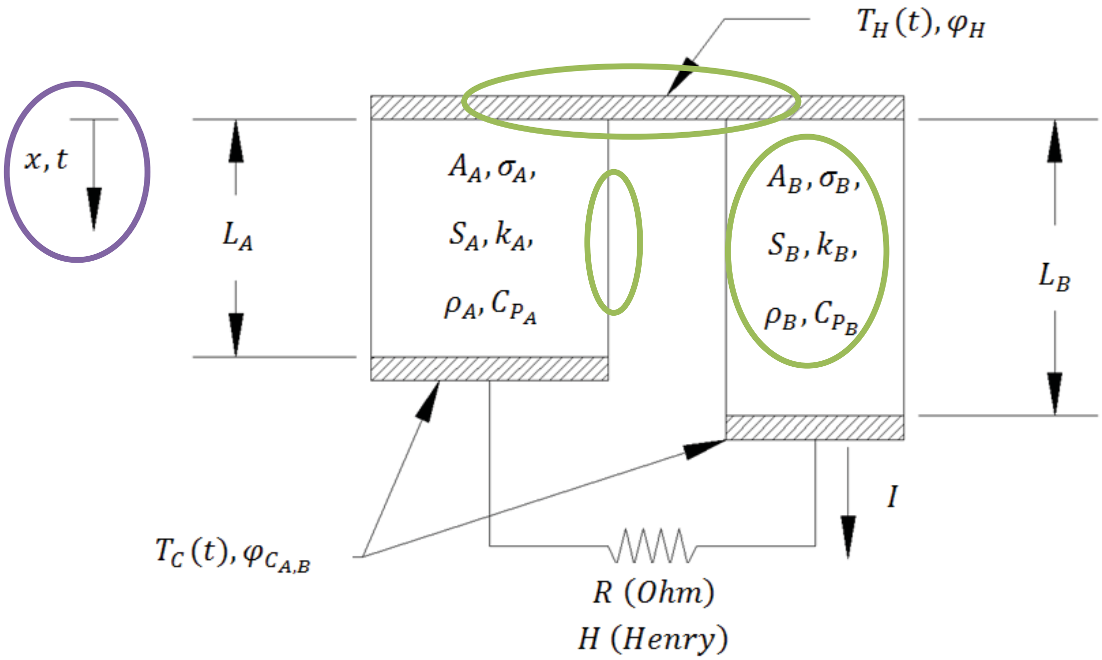
$$u(x) = \int G(x,\xi)f(\xi)d\xi$$

Transient Parameters

Thermal diffusivity factor- $$\Gamma_{A,B} = \frac{\alpha_{avg}L_{A,B}^2}{\alpha_{A,B}L_{avg}^2}$$

Inductance factor- $$\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$$

Transient Model

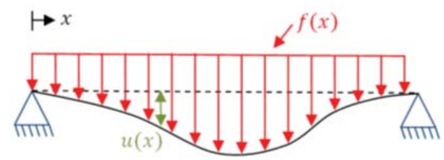


Thermal- $$\frac{\partial}{\partial x}\left[-k_{A,B}\frac{\partial T_{A,B}}{\partial x}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{\partial T_{A,B}}{\partial x} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = \boxed{\rho_{A,B} c_{p_{A,B}} \frac{\partial T_{A,B}}{\partial t}}$$

Electrical- $$\frac{\partial \varphi_{A,B}}{\partial x} = -S_{A,B}\frac{\partial T_{A,B}}{\partial x} - \frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$$

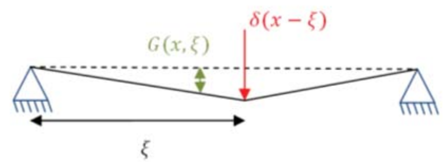
System- $$\varphi_B(L_B) - \varphi_A(L_A) = IR + \boxed{H\frac{dI}{dt}}$$

Green's Function Solution



$$\mathcal{L}u(x) = f(x)$$

$$\mathcal{L}^*G(x,\xi) = \delta(x-\xi)$$



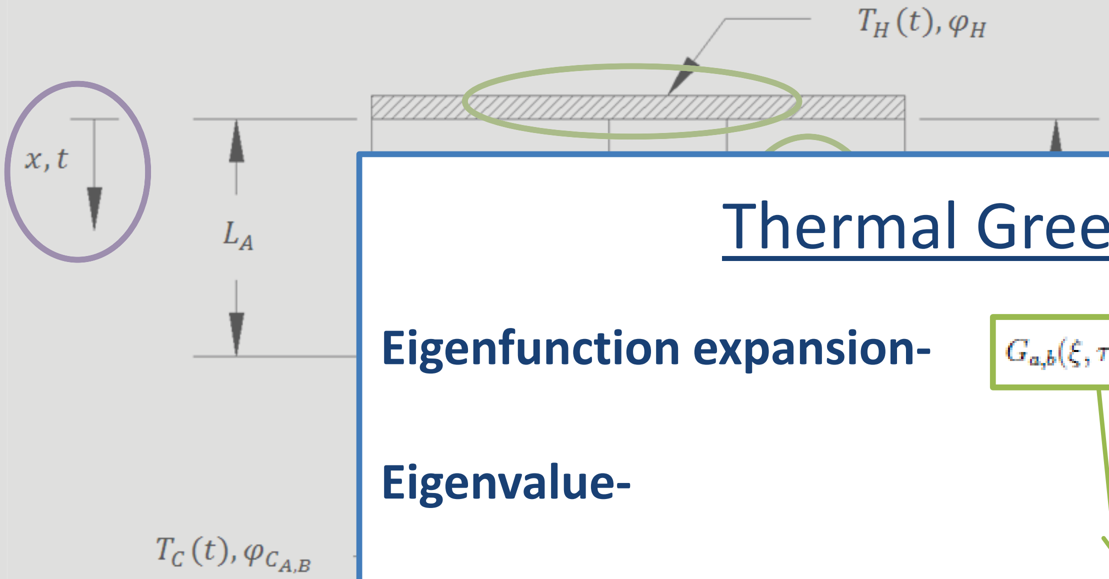
$$u(x) = \int G(x,\xi)f(\xi)d\xi$$

Transient Parameters

Thermal diffusivity factor- $$\Gamma_{A,B} = \frac{\alpha_{avg}L_{A,B}^2}{\alpha_{A,B}L_{avg}^2}$$

Inductance factor- $$\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$$

Transient Model

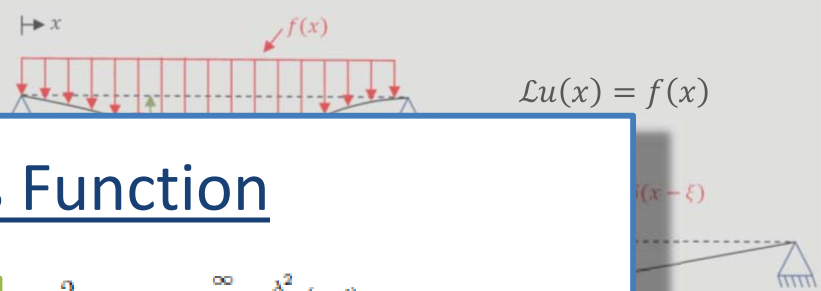


Thermal- $\frac{\partial}{\partial x}\Big[-$

Electrical- $\frac{\partial \varphi_{A,B}}{\partial x} = -S_{A,B}\frac{\partial T_{A,B}}{\partial x} - \frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$

System- $\varphi_B(L_B) - \varphi_A(L_A) = IR + \boxed{H\frac{dI}{dt}}$

Green's Function Solution



$\mathcal{L}u(x) = f(x)$

...ers

factor- $\Gamma_{A,B} = \frac{\alpha_{avg} L_{A,B}^2}{\alpha_{A,B} L_{avg}^2}$

Inductance factor- $\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$

Thermal Green's Function

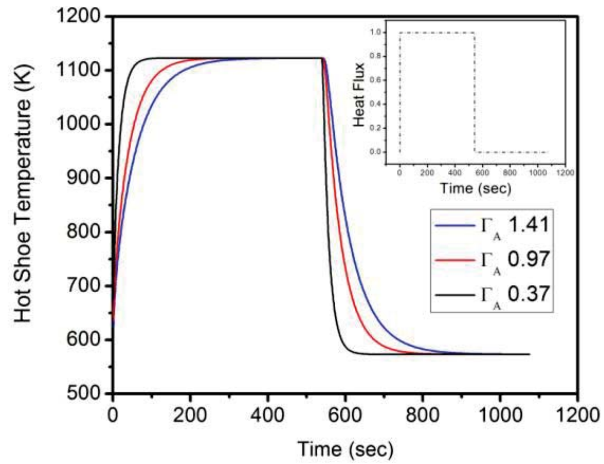
Eigenfunction expansion- $\boxed{G_{a,b}(\xi,\tau;x,t)} = \frac{-2}{\Gamma_{a,b}} H(t-\tau) \sum_{n=0}^{\infty} e^{\frac{\lambda_n^2}{\Gamma_{a,b}}(\tau - t)} \cos(\lambda_n x)\cos(\lambda_n \xi),$

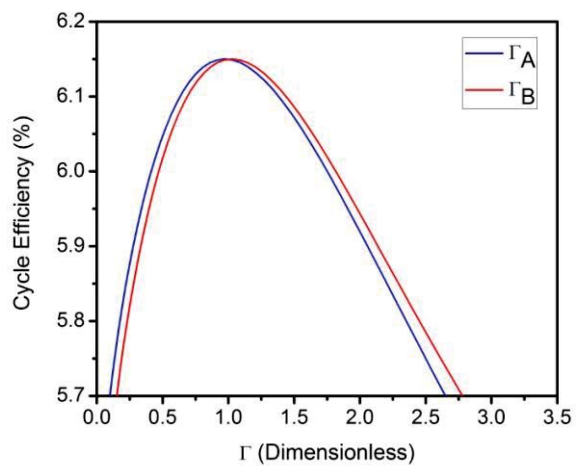
Eigenvalue- $\lambda_n = \frac{(2n+1)\pi}{2},$

Temperature-

$$\hat{T}_{a,b}(x,t) = \boxed{-\Gamma_{a,b}\int_0^1 G_{a,b}(\xi,0;x,t)T_i d\xi} + \boxed{\int_0^t G_{\xi_{a,b}}(1,\tau;x,t)T_c d\tau}$$
$$\boxed{-\int_0^t G_{a,b}(0,\tau;x,t)(A\sin(\omega\tau)+B)d\tau} - \boxed{\int_0^t \int_0^1 G_{a,b}(\xi,\tau;x,t)\gamma I^2(\tau)d\xi d\tau.}$$

Periodic On/Off Operation





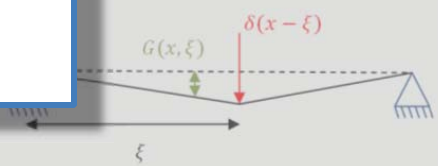
Design Guideline

$$\frac{L_A}{L_B} = \frac{\sqrt{2a}+1}{2a-1}$$

$$a = 1 + \frac{\alpha_B}{\alpha_A}$$

...on Solution

$$\mathcal{L}u(x) = f(x)$$



$$u(x) = \int G(x,\xi) f(\xi) d\xi$$

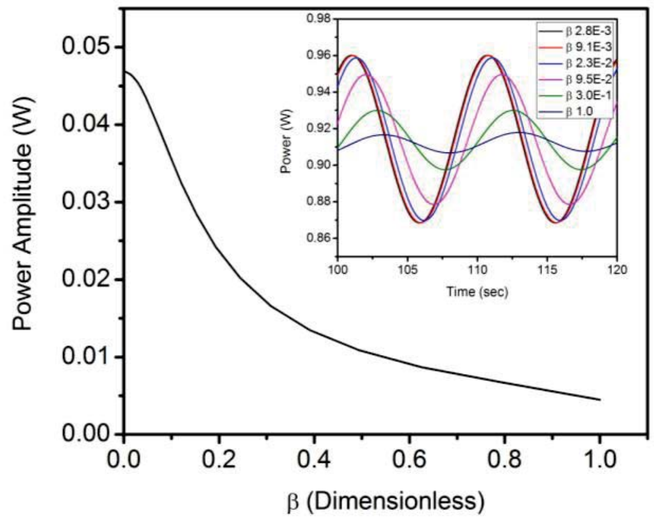
Transient Parameters

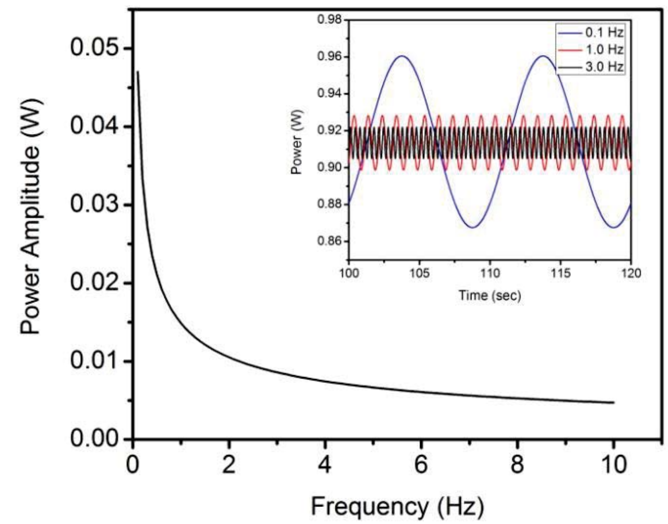
Thermal diffusivity factor- $\Gamma_{A,B} = \frac{\alpha_{avg} L_{A,B}^2}{\alpha_{A,B} L_{avg}^2}$

Inductance factor- $\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$

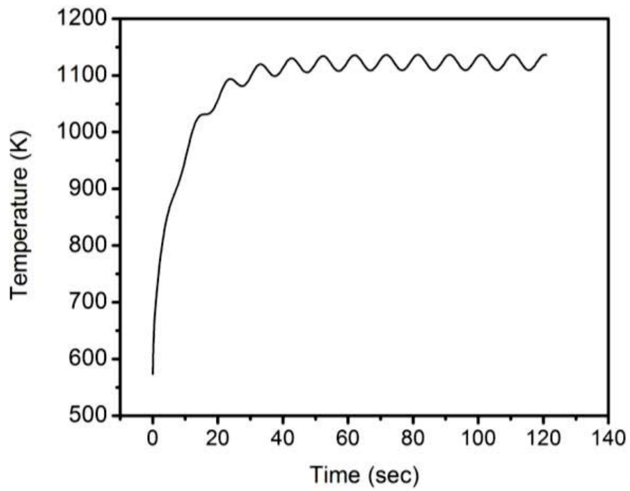
$$\varphi_B(L_B) - \varphi_A(L_A) = IR + H\frac{dI}{dt}$$

Power Output Amplitude





Sinusoidal Operation



Transient Parameters

Thermal diffusivity factor-

t =RC #A$ #A$ L t #A$ =RC

Inductance factor-

* =RC L 4 t =RC

Conclusion

- Asymptotic expansions are an effective means of understanding thermocouple behavior.
- Conversion efficiency is most sensitive to thermal conductivity temperature dependence.
- Thermal diffusivity factor
  - Governs transient operation of a thermocouple, with an ideal value of unity.
- Inductance factor
  - Governs the balance between thermal and electrical inductance.

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JPL

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